

should be recognized that the correlation requires satisfying, with least-square errors,  $I(I-1)/2$  simultaneous relations for  $I$  coefficients, not an isolated equation such as that examined in the comment.

Professor Libby questions "the need for the approximation when detailed calculations of boundary-layer flows involving multicomponent diffusion are being performed." It would seem that the primary consideration here is the tradeoff between accuracy and computational conveniences (in particular, speed, storage, and input requirements). Although not having performed large-scale boundary-layer computations utilizing the approach suggested by Professor Libby, our experience to date leads us to believe that substantial computation convenience is achieved with little loss of accuracy. The boundary-layer equations proposed in our paper have recently been programmed<sup>2</sup> and solutions have been obtained considering 30 molecular species.<sup>3</sup> Philco 212 computer time required for a nonsimilar boundary-layer solution over an ablating body in this case was 6 min. This included the evaluation of edge conditions, a similar solution at the stagnation point, and nonsimilar solutions at 19 additional stations. However, as pointed out in our paper, the primary motivation for introducing the approximation was to develop a transfer-coefficient model to be used in lieu of large-scale boundary-layer computations. In constructing a film coefficient model considering unequal diffusion effects, a convenient alternative approach of equivalent accuracy does not exist.

#### References

- <sup>1</sup> Bartlett, E. P., Kendall, R. M., and Rindal, R. A., "A Unified Approximation for Mixture Transport Properties for Multicomponent Boundary-Layer Applications," Rept. 66-7, Part IV, March 1967, Aerotherm Corp.
- <sup>2</sup> Kendall, R. M. and Bartlett, E. P., "Nonsimilar Solution of the Multicomponent Laminar Boundary Layer by an Integral Matrix Method," Paper 67-218, Jan. 1967, AIAA; also, Rept. 66-7, Part III, March 1967, Aerotherm Corp.
- <sup>3</sup> Kendall, R. M., Bartlett, E. P., Rindal, R. A., and Moyer, C. B., "An Analysis of the Coupled Chemically Reacting Boundary Layer and Charring Ablator" Rept. 66-7, Part I, March 1967, Aerotherm Corp.

### Comment on "Dynamics of a Radiating Gas with Application to Flow Over a Wavy Wall"

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IN his excellent exposition of the application of the spherical-harmonic approximation to the radiation-transport equation, Cheng found [Ref. 1, Eq. (6.11)] the velocity potential for the linearized flow of a nonscattering grey gas over a small-amplitude wavy wall to be

$$\begin{aligned}\varphi(x_1, x_2) &= \frac{U_\infty l}{2\pi} \operatorname{Re} \left\{ \sum_{j=1}^2 A_j \exp \left[ \frac{2\pi}{l} (c_j x_2 + ix_1) \right] \right\} \\ &= U_\infty \epsilon \sum_{j=1}^2 e^{-2\pi \delta_j (x_2/l)} \left[ a_j \cos 2\pi \left( \frac{x_1 - \lambda_j x_2}{l} \right) - b_j \sin 2\pi \left( \frac{x_1 - \lambda_j x_2}{l} \right) \right]\end{aligned}$$

where the wall shape was given by

$$X_2 = \epsilon \sin 2\pi x_1/l$$

and  $A_j$ ,  $c_j$ ,  $\delta_j$ ,  $\lambda_j$ ,  $a_j$ , and  $b_j$  could be found from his Eqs. (6.8-6.10).

From this velocity potential, it should follow that the normal velocity perturbation may be written as

$$\begin{aligned}\frac{u_2'}{U_\infty} &= \frac{1}{U_\infty} \frac{\partial \varphi}{\partial x_2} = -\frac{2\pi \epsilon}{l} \sum_{j=1}^2 e^{-2\pi \delta_j (x_2/l)} \times \\ &\quad \left\{ (a_j \delta_j - b_j \lambda_j) \cos \frac{2\pi}{l} (x_1 - \lambda_j x_2) - (a_j \lambda_j + b_j \delta_j) \sin \frac{2\pi}{l} (x_1 - \lambda_j x_2) \right\}\end{aligned}$$

However, Cheng, in writing this expression in his Eq. (6.13b), included only the first term; the second, sine, term was omitted. In addition, through a typographical error, the coefficient of the cosine term was incorrectly given as  $(a_j \delta_j + b_j \lambda_j)$ .

It should be noted that where  $x_2 = 0$ , the coefficient of the sine term is identically zero. This is a consequence of Cheng's Eq. (6.9):

$$\sum_{j=1}^2 A_j c_j = 2\pi \frac{\epsilon}{l}$$

Thus, on the wall surface, the omitted term equals zero. However, since Cheng's solution should be valid throughout the flow, the omitted sine term must be included in any calculation of  $u_2'$  for  $x_2$  not identically zero.

#### Reference

- <sup>1</sup> Cheng, P., "Dynamics of a Radiating Gas with Application to Flow Over a Wavy Wall," *AIAA Journal*, Vol. 4, No. 2, 1966, pp. 238-245.

### Comment on "Computation of Stress Resultants from the Element Stiffness Matrices"

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THE general form of Eq. (1), Ref. 1, to take into account not only continuous distributed loads, but also discontinuous and concentrated loads applied outside the nodes, was presented in a previous work.<sup>2</sup> The partitioning of Eq. (5) of Ref. 2 relative to the structural elements of the structural system results, for each element, in an equation exactly like Eq. (3), Ref. 1 (with the exception of the inertia term which was not considered in Ref. 2). The generalized forces  $\{Q_i\}$  are precisely the component submatrices of  $[s]$  [Eq. (5) of Ref. 2]. The generalized forces  $\{Q_i\}$  of the second equation of the example are simply the fixed-end bending moments and shear forces of the elements. This concept was already introduced in Refs. 2 and 3.

From the final results of the example and the final paragraph of the Note a misleading conclusion may be drawn that the method discussed is approximate. In reality the method is theoretically an exact one as the generalized displacements are obtained from the equilibrium equations relative to the nodes, and these are exactly satisfied (except for computational er-

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